Handling Uncertainty:
Recent Perspectives in Uncertainty Quantification Methods for Differential Equations

Giulia Bertaglia*

* Department of Environmental and Prevention Sciences
University of Ferrara
C.so Ercole I D'Este 32, 44121 Ferrara, Italy
e-mail: giulia.bertaglia@unife.it, web page: http://giuliabertagliaphd.wordpress.com

ABSTRACT

Scientific computing has become a central tool across many research fields for analysing the dynamics of complex systems. This is especially true in engineering and biological applications, where experimental studies are often time-consuming, costly, and difficult to replicate. Traditionally, mathematical models are solved using deterministic numerical techniques under the assumption that initial conditions, boundary conditions, and all model parameters are precisely known. However, this assumption is rarely satisfied in practice due to limitations in measurement capabilities, observational biases, and incomplete understanding of underlying processes. In the presence of such uncertainties, it becomes essential to quantify their impact to interpret simulations meaningfully. In recent years, this need has driven a growing interest in the development of uncertainty quantification (UQ) techniques for systems governed by differential equations. Among these, Monte Carlo methods remain the simplest and most widely used approach, but their slow convergence often leads to high computational costs especially when each sample requires solving complex systems. To address this challenge, two main strategies have emerged: the use of surrogate models to replace costly evaluations with cheaper approximations while reducing the variance of the estimator [1,3], and methods based on stochastic orthogonal polynomials that accelerate convergence rates by exploiting solution regularity [2,4]. In this talk, I will present and compare recent advances in numerical strategies for UQ, including bi-fidelity methods and multi-order Monte Carlo approaches, discussing their advantages and limitations through practical examples with a focus on biomathematical applications. Numerical experiments will illustrate the critical role of advanced numerical methods in translating mathematical models into meaningful and reliable insights even when dealing with complex systems under uncertainty.

REFERENCES

[1] G. Bertaglia, W. Boscheri, L. Pareschi, Multi-Order Monte Carlo approaches leveraging hierarchies of IMEX schemes to quantify uncertainties in multiscale systems. Preprint (2025).

[2] G. Bertaglia, V. Caleffi, L. Pareschi, A. Valiani, Uncertainty quantification of viscoelastic parameters in arterial hemodynamics with the a-FSI blood flow model. J. Comput. Phys. (2021) 430:110102.

[3] G. Bertaglia, L. Liu, L. Pareschi, X. Zhu, Bi-fidelity stochastic collocation methods for epidemic transport models with uncertainties. Netw. Heterog. Media (2022) 17(3):401-425.

[4] G. Bertaglia, L. Pareschi, Hyperbolic compartmental models for epidemic spread on networks with uncertain data: application to the emergence of Covid-19 in Italy. Math. Models Methods Appl. Sci. (2021) 31(12):2495-2531.