

PHYSICS AND STRUCTURE-PRESERVING NUMERICAL METHODS FOR VARIATIONAL PROBLEMS

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ABSTRACT

Many variational problems in science and engineering are governed by constraints that encode essential physics. Equality constraints, such as incompressibility, and conservation of energy or momentum, underpin structure-preserving discretizations. Inequality constraints are equally pervasive, arising in free boundary problems, ice-sheet flow, groundwater filtration, and mathematical finance, as well as in regularization-based shock capturing for hyperbolic conservation laws. A unifying perspective is the geometric treatment of the structure governing the underlying physics, spanning conservation laws, exact sequences, and the nonsmoothness of contact, obstacles, and shocks. Designing high-order schemes that preserve structure, or resolve sharp features, with mesh-independent convergence remains an active research area. This minisymposium brings together researchers at the forefront of algorithmic and theoretical developments in physics- and structure-preserving numerical methods for variational problems, with an emphasis on: (a) compatible discretizations, optimization algorithms to achieve mesh independence, structure and constraint preservation, and sharp-feature resolution; (b) a priori and a posteriori error estimates; and (c) applications in contact mechanics, geophysics, electromagnetics, topology optimization, compressible fluid dynamics, and other areas. The session aims to identify open problems including adaptive high-order methods, efficient solvers for

large-scale constrained problems, and the coupling of constrained and regularized variational formulations with other physical models.

SPEAKERS

1. Alex Kaltenbach (Technical University of Berlin, Germany): Duality-Based Numerical Analysis for Nonsmooth Convex Variational Problems
2. Frederic Marazzato (University of Nevada Las Vegas): Proximal Galerkin for variational problems with manifold constraints: theory and examples
3. Alexandre Ern (INRIA/ENPC): Proximal Discontinuous Galerkin Methods for Variational Inequalities
4. Johannes Haubner (University of Graz): Optical flow based optimal control for image registration
5. Robert Kirby (Baylor University): TBD
6. Xiangxiong Zhang (Purdue University, USA): Invariant domain preserving schemes for information geometric regularization of the compressible Euler equations
7. Carlo Lovadina (University of Milan): Mixed Virtual Element Methods for elastic contact problems in the nearly incompressible regime
8. Brian Tran (California State University, Long Beach): Information geometric regularization of Hamilton—Jacobi equations
9. Dohyun Kim (Brown University): Conservative convex separable approximation with Bregman divergence
10. Rami Masri (The Pennsylvania State University): A priori and a posteriori error estimates for the proximal Galerkin method
11. Noe Reyes (Brown University): Stabilized proximal Galerkin methods
12. Hojun Qin (Brown University): Analysis of Bregman Proximal Point for the Obstacle Problem.
13. Alexey Izmailov (Brown University): Convex Splitting Algorithms for Proximal Galerkin Methods
14. Brendan Keith (Brown University): A proximal Galerkin method for large bending isometries

15. Boris Andrews (University of Oxford): Enstrophy-stable discretisation of the incompressible Navier–Stokes equations