

HIGH ORDER STRUCTURE-PRESERVING SCHEMES FOR MULTI-SCALE PDE SYSTEMS: INVOLUTIONS, THERMODYNAMIC COMPATIBILITY AND ASYMPTOTIC LIMITS

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ABSTRACT

Many systems of partial differential equations arising in physics and engineering are endowed with intrinsic geometric, analytic, and thermodynamic structures. These include involution constraints, such as the divergence-free condition of magnetic fields or the curl-free character of deformation tensor, thermodynamic compatibility requirements encoded in energy conservation and entropy balance or inequalities, and physical admissibility constraints such as positivity of density and pressure.

At the same time, these PDE systems are often genuinely multiscale. Important examples include low-Mach-number flows, stiff relaxation systems, and hyperbolic reformulations of parabolic models, where the correct limiting behavior must be recovered in asymptotic regimes. These features make the design of reliable numerical methods particularly demanding: schemes should be accurate and robust, while preserving, at the discrete level, the fundamental structural properties of the continuous problem.

The aim of this minisymposium is to review recent advances in high order structure-preserving methods for multiscale PDE systems, with emphasis on the preservation of involutions, thermodynamic compatibility, asymptotic limits, and physical admissibility constraints. Contributions will cover both methodological developments and applications, including continuum mechanics, plasma flows, electromagnetics, and multi-material flows.

SPEAKERS

1. Andrea Thomann (Inria Strasbourg, FR)
Relaxed schemes for hyperbolic systems
2. Rémi Abgrall and Mario Ricchiuto (INRIA Bordeaux, FR)
Active flux and involution preservation
3. Sara Rinaldi (Univ. of Trento, IT)
Semi-implicit thermodynamically compatible schemes
4. Christiane Helzel (Univ. of Düsseldorf, GE)
Third order accurate wave propagation methods with adaptive mesh refinement
5. Mária Lukáčová-Medvidová (Univ. of Mainz, GE)
TBA
6. Gabriella Puppo (Univ. La Sapienza, IT)
Low Mach schemes for hyperbolic systems
7. Lorenzo Micalizzi (North Carolina State University, USA)
Dual Formulation Methods for Hyperbolic Conservation Laws
8. Maurizio Tavelli (Univ. of Verona, IT)
Novel semi-implicit solvers for the shallow water equations in covariant form
9. Kailiang Wu (SUSTech, China)
The Hidden Unity of Nonlinear Stability: A Weak-to-Strong Lifting Framework for Entropy Stability, Bound Preservation, and Oscillation Control
10. Eric Sonnendrücker (Max-Planck Institut, GE)
Metriplectic numerical methods for the Vlasov-Maxwell equations with collisions
11. Domenico Caparello (Univ. of Ferrara, IT)
Hierarchical Dynamic Domain Decomposition Methods for the Boltzmann Equation
12. Christian Klingenberg (Univ. of Würzburg, GE)
How good does Active Flux work in practice compared to other methods?
13. Michael Dumbser (Univ. of Trento, IT)
A new family of structure-preserving discontinuous Galerkin schemes for hyperbolic PDE with involutions

14. Laura Del Rio (INRIA Bordeaux, FR)
A structure-preserving method for a new model of multiphase flows with dispersive regularization
15. Matteo Semplice (Univ. of Insubria, IT)
Weights based involution-preserving discretizations
16. Enrico Zampa (Univ. of Wien, A)
A fully discrete, arbitrary high order, involution-preserving and provably locally energy/entropy stable space-time method for a class of nonlinear wave systems