

MATERIAL ROBUSTNESS: STRUCTURE-PRESERVING DISCRETIZATIONS FOR SOLIDS AND COMPLEX FLUIDS

UMBERTO ZERBINATI*, CHARLES PARKER[†] AND PABLO
BRUBECK[†]

*Simula Research Laboratory, Kristian Augusts gate 23, 0164 Oslo, Norway
umberto@simula.no (Corresponding Organizer), <https://www.uzerbinati.eu>

[†]Mathematical Institute, University of Oxford, Woodstock Road, Oxford OX2 6GG, UK
parker@maths.ox.ac.uk, brubeck@maths.ox.ac.uk

ABSTRACT

Many variational problems in continuum mechanics, including the Hellinger–Reissner formulation of linear elasticity, the velocity–stress formulations of incompressible and complex flows, and their magnetohydrodynamic and liquid-crystalline counterparts, are governed by a stress tensor whose algebraic structure is dictated by the balance of angular momentum. In the classical setting this manifests as the symmetry of the Cauchy stress, while in polar and micropolar media it manifests as a controlled non-symmetry.

Recent work [1] has shown that whether a discretization respects this structure exactly, rather than approximately, may drastically affect the accuracy of the computed stress: schemes enforcing the structure only weakly may return arbitrarily poor stress approximations under anisotropic constitutive laws, even in zero-stress configurations, whereas schemes enforcing it strongly deliver stress errors independent of the constitutive law. We have termed this property *material robustness*, in analogy with pressure robustness in incompressible flow [2].

Strongly symmetric elements often come in two flavors: very low-order, or impractically high-order with supersmoothness. Constructing intermediate, but still high-order, spaces and appropriate solvers remains an active area of research that some of the talks will address. Moreover, circumventing the symmetry constraint via weak imposition also requires high-order methods to obtain accurate solutions for some constitutive laws.

This minisymposium gathers recent advances on material robustness across theory, implementation, methodology, and applications: the unifying theory and its finite element tensor calculus interpretation; high-order frameworks such as TDNNS and

Trefftz methods, and the solvers they require; and applications to thin domains, liquid crystals, magnetohydrodynamic equilibria, and complex fluids. Together, the talks position structure preservation as a unifying design principle for accurate, reliable, high-order discretizations throughout continuum mechanics. We envisage two to three sessions.

SPEAKERS

1. Charles Parker (University of Oxford): Material robustness: motivation and a unifying theory.
2. Kaibo Hu (University of Oxford): A finite element tensor calculus perspective for microstructure parameter-robustness.
3. Boris Andrews (University of Oxford): Material robustness of strongly symmetric stress discretizations in thin domains.
4. Enrico Zampa (University of Vienna): Material robustness of the TDNNS method for the Stokes problem.
5. Paul Stocker (University of Vienna): Trefftz methods for material-robust discretizations.
6. Pablo Brubeck (University of Oxford): Solvers and preconditioners for strongly symmetric stress discretizations.
7. Jingmin Xia (National University of Defense Technology): Material robustness in geometrically frustrated smectics: a three-field mixed finite element approach.
8. Thomas Higham (University of Oxford): Material robustness in MHD equilibria.
9. Umberto Zerbinati (Simula Research Laboratory): Material robustness for complex fluids with non-symmetric Cauchy stress.

REFERENCES

- [1] P. Brubeck, C. Parker, and U. Zerbinati. Achieving material robustness via symmetric stress finite element discretizations. *Computer Methods in Applied Mechanics and Engineering*, 461:119262, 2026.
- [2] V. John, A. Linke, C. Merdon, M. Neilan, and L. G. Rebholz. On the divergence constraint in mixed finite element methods for incompressible flows. *SIAM Review*, 59(3):492–544, 2017.