

MINIMUM-RESIDUAL METHODS AND T-COERCIVITY: RECENT ADVANCES AND EMERGING CONNECTIONS

FLEUR BERTRAND*, PHILIPP BRINGMANN[†], ERELL JAMELOT[‡]

*Technische Universität Chemnitz
Reichenhainer Straße 41, 09126 Chemnitz, Germany
fleurianne.bertrand@mathematik.tu-chemnitz.de

[†]TU Wien, Institute of Analysis and Scientific Computing
Wiedner Hauptstraße 8–10, 1040 Wien, Austria
philipp.bringmann@asc.tuwien.ac.at

[‡]Université Paris-Saclay, CEA, STMF
91191 Gif-sur-Yvette cedex, France
erell.jamelot@cea.fr

ABSTRACT

Many partial differential equations—including those with indefinite or non-selfadjoint operators or parameter-dependent stability estimates—are not uniformly coercive. Examples occur in wave propagation, convection-dominated problems, and problems with sign-changing coefficients. In such settings, standard Galerkin methods may fail to preserve the stability of the continuous problem.

Two complementary frameworks address this difficulty. Minimum-residual methods compute an approximation by minimizing the residual in a suitable dual norm. The choice of the test inner product, which determines the associated trial-to-test operator, is crucial for the stability of the method and the quality of the approximation. For linear problems in Hilbert spaces, residual minimization can yield symmetric positive-definite formulations even when the original operator is indefinite.

Classically, the well-posedness of variational forms of linear problems is achieved thanks to the inf-sup condition and the solvability condition. These conditions can be derived with the help of T-coercivity. A bounded, invertible transformation operator is introduced so that the transformed bilinear form becomes coercive. This makes it possible to establish an inf-sup condition or apply Fredholm theory when the original form is not coercive. The transformed problem is equivalent to the original one and stability estimates can be transferred between the two formulations. Both theories share a common principle: they associate trial functions with suitable test functions to establish stability. Minimum-residual methods construct this association through the trial-to-test operator, while T-coercivity uses a transformation

that makes the variational form coercive. This raises the common question: how can these operators be approximated while preserving stability at the discrete level? This minisymposium will bring together experts in both minimum-residual methods and T-coercivity to present their latest research and discuss the connections between these two perspectives. Exploring this connection may allow analytical tools and computational techniques developed in either framework to benefit the other.

SPEAKERS

1. Roland Becker (Université de Pau et des Pays de l'Adour, France): Newton-AFEM for ODEs.
2. Zhiqiang Cai (Purdue University, USA): T-coercive convex optimization problems arising from ReLU neural network approximations.
3. Patrick Ciarlet (ENSTA, Institut Polytechnique de Paris, France): Using explicit T-coercivity to design efficient and inexpensive solvers for diffusion.
4. Wolfgang Dahmen (University of South Carolina, USA): Minimum Residual Methods and Operator Learning.
5. Jay Gopalakrishnan (Portland State University, USA): Tools to prove stability for indefinite problems.
6. Martin Halla (Karlsruhe Institute of Technology, Germany): On the instabilities of naive FEM discretizations for PDEs with sign-changing coefficients.
7. Antti Niemi (University of Oulu, Finland): An Ultra-Weak Minimum Residual Framework for Harmonic Analysis of Shells of Revolution.
8. Maciej Paszynski (AGH University of Krakow, Poland): Kolmogorov-Arnold Neural-Networks with NURBS-based activation functions.
9. Tomasz Słuzalec (AGH University of Krakow, Poland): Minimum-Residual Approaches for Physics-Informed Neural Networks.
10. Gerhard Starke (Universität Duisburg-Essen, Germany): Comparison of minimum residual formulations for elastoplasticity.
11. Subhashree Mohapatra (IIIT Delhi, India): Spectral element method for three dimensional elliptic problems with smooth interfaces.
12. François Madiot (CEA, France) A posteriori error estimates for mixed finite element discretizations of the Neutron Diffusion equations.