

# LEARNING DIFFERENTIAL EQUATIONS FOR SCIENCE AND ENGINEERING APPLICATIONS VIA SCIENTIFIC MACHINE LEARNING

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## ABSTRACT

Many science and engineering applications feature differential equations with terms that are partially unknown. For example, in neural ODEs, one tries to train a neural network to model a dynamical system. Similarly, in (non-intrusive) reduced order models (NIROMs), operators are derived in such a way to closely match (projected) snapshot data. Additionally, recent years have seen a big push to replace empirical and often *ad hoc* subgrid-scale process models (e.g., closure terms in turbulence modelling, constitutive models in structural mechanics, physics process representations in climate models) in high fidelity modelling toolchains with machine learned representations. Many existing approaches try to learn these terms in an ‘offline’ fashion, e.g. through supervised learning, and then substituted back into the differential equation in order to give predictions in an ‘online’ setting [1]. An alternative to this “operator-fitting” approach is known as “embedded model learning”, “solver-in-the-loop”, or “trajectory-fitting” [2,3]. In this approach, one learns a model in such a way that, upon embedding in the solver, it results in accurate predictions of the solution trajectory. This has the promise to lead to more stable models, but comes at the price of increased computational costs associated with differentiating through the entire differential equation solver (e.g. by using fully differentiable solvers or adjoints). Such fully differentiable solvers are actively being developed in the Scientific Machine Learning community [4].

In this minisymposium we bring together researchers working on learning models for various science and engineering applications (e.g., computational fluid mechanics, structural modelling, climate modelling), either with operator fitting or with embedded learning. We welcome contributions on the topic of learning turbulence models, reduced order models, and other types of ‘closure’ models that appear in partial differential equations.

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