9th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMA2024)
June 3-7, 2024, Lisbon, Portugal

NUMERICAL METHODS FOR EULER FLOWS: DISSIPATIVE WEAK SOLUTIONS AND SINGULAR BEHAVIOR (800)

NIKLAS FEHN[*] AND MARTIN KRONBICHLER[*]

[*] University of Augsburg, Germany
Universitätsstraße 12a, 86159 Augsburg
niklas.fehn@uni-a.de, martin.kronbichler@uni-a.de

Key words: Euler, Fluid Dynamics, Hyperbolic Transport, Onsager, Singularities, Turbulence.

ABSTRACT

The dynamics of nonlinear hyperbolic transport in fluid dynamics fascinate not only mathematicians and physicists due to the open Clay Millennium Navier-Stokes problem or the hidden mysteries of turbulence, but also CFD method developers and engineers due to the highly nonlinear nature of the model problem involving complex, fine-scale physical processes and the need for robust, computationally efficient numerical methods.

For the compressible Euler equations, singularities in the form of shocks are well understood. However, also in the incompressible or low-Ma regime, the Euler equations may develop singularities. Turbulent flows are characterized by a rich spectrum of scales and a positive dissipation rate even in the limit of vanishing viscosity (infinite Reynolds number), a phenomenon termed anomalous energy dissipation or the zeroth law of turbulence [1].

Onsager conjectured that the occurrence of anomalous dissipation is linked to singular behavior, i.e. a continuous velocity field exhibiting infinite gradients (precisely, Hölder continuity with exponent <1/3). While this statement remained a conjecture for almost a century, Onsager's hypothesis has been proven recently, see e.g. [2]. This novel mathematical insight might open new paradigms in the development of numerical methods for Euler (or high-Re Navier-Stokes) flows, e.g. by taking into account the mathematical regularity of the problem (involving singular behavior) in the analysis and design of a discretization scheme.

This mini-symposium aims to bring together researchers from various disciplines engaged in the fields of Euler equations, Onsager's conjecture, turbulence, or large-eddy simulation. Due to the interdisciplinarity of the topic, we invite contributions with a numerical but also mathematical focus, from the compressible or incompressible Navier-Stokes/Euler communities. An open problem is to prove convergence of a numerical scheme to dissipative weak solutions. Emerging structure-preserving or physics-compatible numerical methods aiming at low-dissipation, scale-resolving simulations face the challenge to treat anomalous dissipation and singularities in a generic, numerically robust, and physically consistent way, see e.g. [3].

REFERENCES

[1] B. Dubrulle, Beyond Kolmogorov cascades, *Journal of Fluid Mechanics* 867, P1, 2019.

[2] P. Isett, A proof of Onsager's conjecture, *Annals of Mathematics* 188, 871–963, 2018.

[3] N. Fehn, M. Kronbichler, P. Munch, and W. A. Wall, Numerical evidence of anomalous energy dissipation in incompressible Euler flows: towards grid-converged results for the inviscid Taylor-Green problem, *Journal of Fluid Mechanics* 932, A40, 2022.