

## DIFFERENTIAL PROBLEMS ON HETEROGENEOUS STRUCTURES AND NETWORKS: MODELLING, NUMERICS AND APPLICATIONS

TRACK NUMBER (100 TO 2200 AS EXPLAINED ON THE WEB SITE)

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### ABSTRACT

Numerous physical phenomena are modelled in unconventional geometries, encompassing ramified spaces, unions or intersections of manifolds, fractals, networks, etc.

One example is given by transport and growth models, where interconnected structures such as roads, pipelines, and vessel networks serve as domains for both linear and nonlinear partial differential equations. These equations, often hyperbolic or hyperbolic-dominant, exhibit low degrees of regularity, evident for example in Conservation Laws and Hamilton-Jacobi equations. This limited regularity is further complicated by the non-Euclidean structure of the domain. Consequently, the treatment of such equations requires a focused understanding of solutions and their numerical processing.

Another example can be found in the modeling of flow, transport and mechanical deformation processes in porous fractured media. Fractures are thin structures embedded in a much larger domain, such that their geometrical reduction to planar interfaces is of great importance to reduce the computational cost of simulations. After such dimensional reduction, the problem is rewritten as a system of coupled PDEs on a hybrid dimensional domain: the 3D bulk domain, the 2D fractures and the 1D fracture intersections.

In some applications (e.g., the growth of roots in soil, the formation of new blood vessels in tissues,...) a direct coupling of a 3D problem with a 1D problem might appear as a result of geometrical reduction. This is the case, for example, of thin and elongated structures embedded in a 3D domain, that can be conveniently reduced to line interfaces. In this case, however, the geometrical simplification results in an ill-posed mathematical problem in the standard approximation spaces and ad-hoc formulations and numerical schemes need to be developed.

The principal aim of this mini-symposium is to convene international experts in this field, providing a platform for the presentation of the most recent advances and fostering discussions on the current state of the art in overcoming these challenges.