REDUCED-ORDER MODELING OF EXPERIMENTAL TURBULENT FLOWS: FROM LINEAR PROJECTION-BASED METHODS TO AUTOENCODERS

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ABSTRACT

Turbulent flows, a longstanding challenge in classical physics, are characterized by their nonlinear, multi-scale, and chaotic behavior. While the Navier-Stokes equations, coupled with the thermodynamic equation for the fluid state, provide a comprehensive mathematical foundation, solving them numerically remains computationally expensive. This poses a challenge for iterative applications, such as closed-loop control or design optimization, which require surrogate models for faster simulations. In this paper, reduced-order models (ROMs) are explored as a promising solution. ROMs are techniques designed to capture the essential dynamics of turbulent flows at a significantly reduced computational cost by identifying and exploiting features inherent in the flow field [1]. Two main classes of ROMs can be defined: projection-based methods and operator-based methods. Projection-based models, while effective, are inherently intrusive, as they require the application of basis expansions and projections onto the full-order model's operators. In contrast, operator-based ROMs offer nonintrusive alternatives but depend on prior knowledge of the ROM structure. To address the limitations of both approaches, this work explores the potential of neural compression algorithms, i.e. the application of neural networks for unsupervised feature extraction to compress data into lower-dimensional representations.

Recent studies by Fukami et al. [2], Agostini [3], Eivazi et al. [4] and Solera-Rico et al. [5] have demonstrated the superior performance of convolutional neural network-based standard autoencoders (AE) and variational auto encoders (VAE) over traditional projection-based methods, such as Proper Orthogonal Decomposition (POD). However, further research is needed to evaluate the performance of these methods on experimental datasets characterized by diverse flow regimes and to improve the interpretability of the transformations learned by these methods. As a proof of concept, this study examines the flow developing around a one degree-of-freedom elastically-mounted cylinder experiencing vortex-induced vibrations [6]. The dataset consists of time-resolved, two-component velocity snapshots obtained experimentally by Particle Image Velocimetry for different Reynolds numbers, ranging from 6000 to 16000,

based on the cylinder diameter D and the free-stream velocity U_{∞} , in the wake of the oscillating cylinder. This work makes several contributions: (i) it assesses the performance of both AE and VAE compared to POD in reconstructing unseen flow regimes, as shown in Fig. 1; (ii) it develops a single model capable of representing all Reynolds numbers in the dataset with just three variables; and (iii) it proposes novel techniques to analyze and exploit the mapping learned by the neural network, as presented in Fig. 2.



Fig.1: Measured fluctuation velocity field and reconstructions for a single time sample at the highest Reynolds number not seen during training. The flow field is measured using planar two-component particle image velocimetry. The reduced free-stream velocity is $U^* = U_{\infty}/(f_n D) = 12.64$, corresponding to a Reynolds number Re = 13826, based on the cylinder diameter D and the free-stream velocity U_{∞} . Reconstructions are shown for proper orthogonal decomposition (POD) using three modes, and for both the standard autoencoder (AE) and variational autoencoder (VAE), with latent space dimensions set to three.



Fig. 2: "Latent-Space Walk". On the left, the three-dimensional latent space encoded by the variational autoencoder (VAE) is projected onto a 2D plane using UMAP [7]. Each point represents a time snapshot of different vortex-induced vibration regimes, color-coded accordingly. The cross marks an equilateral triangle grid used to sample points that are fed into the VAE decoder to generate new data fields, displayed on the right. The decoder's output illustrates how the VAE interprets transitions between different regimes. The grid axes are shown to help correlate specific points in the latent space (left) with their corresponding reconstructed fields (right). For instance, the sample point closest to the origin of the grid on the left corresponds to the bottom-left image on the right.

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