Fast Solution Techniques for Polytopic *hp*-Discontinuous Galerkin Methods for Radiation Transport Problems

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ABSTRACT

In this talk we consider the application of *hp*–version discontinuous Galerkin finite element methods (DGFEMs) for the discretization of the radiation transport problem on general (spatial) computational meshes consisting of polytopic elements. Our particular interest is the application to medical treatment planning in clinical radiotherapy. Here we study both the stability and a priori error analysis of the proposed scheme. The implementation is based on exploiting a nodal approximation in energy and angle, together with fast numerical integration techniques on the spatial polytopic mesh; this approach leads to a highly parallelisable algorithm whereby a large number of linear transport solves must be computed. Additionally, we consider the design of iterative solution techniques for the resulting linear system of equations; in particular, we consider both a Richardson type iteration, as well as a Richardson preconditioned GMRES solver for both monoenergetic and polyenergetic problems. For the Richardson scheme, we prove convergence, independent of the discretisation parameters h and p, while GMRES is shown (computationally) to depend only mildly on h and p. Numerical experiments are presented to highlight the accuracy and performance of the proposed solver, as well as to benchmark with more standard kinetic Monte Carlo simulations.

REFERENCES

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