Reduced-Order Models in Bayesian solvers for inverse problems

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ABSTRACT

Challenging inverse problems aim at identifying large sets of parameters using data from different sources and diverse accuracy. This is the case of data assimilation for geophysical crust dynamics, were the number of parameters to identify amounts to thousands. In this context, Bayesian inverse solvers combined with Markov-Chain Monte Carlo (MCMC) strategies are an affordable strategy, accounting for the uncertainty of the input data and quantifying also uncertainty of the output. Despite the efficiency of the MCMC approach, the direct problem has to be evaluated an extremely large number of times, many (after the burn-in phase) with the input parameters lying in a narrow range. This is the ideal situation for Reduced-Order Models (ROM): many repeated queries to the model corresponding to parameters lying in a limited manifold.

Thus, we aim at applying ROM to large-dimensional parametric forward problems. In this case, it is important optimising the dimensionality reduction technique inherent to the ROM strategy. For instance, Proper Orthogonal Decomposition (POD) is associated with a linear Principal Component Analysis (PCA). PCA is linear in the sense that assumes the reduced-dimension manifold to be Euclidean. We explore using kernel PCA (kPCA) to further reduce the dimension, thus devising a kPOD approach. Different options to select physically inspired kernels, based on the knowledge of the problem under consideration, are discussed. Moreover, the computational strategy to explore the feature space (the reduced-dimensional space) is also discussed.

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