

INVERSE PROBLEMS: THEORETICAL CHALLENGES, NUMERICAL SOLUTIONS, AND CUTTING-EDGE APPLICATIONS IN BIOMEDICINE AND MATERIAL ENGINEERING

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ABSTRACT

Inverse problems are crucial for reconstructing unknown quantities from measurements, with key applications in biomedical imaging, material engineering, and geoscience. Leveraging forward models, often based on partial differential equations, inverse problems drive breakthroughs in medical diagnostics and material analysis. However, these problems are typically ill-posed, meaning solutions may not be unique or stable, making resolution highly challenging. While existence and uniqueness can be ensured by relaxing the problem and selecting solutions with specific properties, achieving stability presents a considerably greater challenge.

In biomedicine, inverse problems underpin imaging techniques like MRI, CT scans, and ultrasound, where indirect data must be transformed into detailed visual representations of tissues, aiding diagnosis and treatment planning. They are also employed to optimise model parameters and therapy schedules in patient-specific image-informed mathematical models of disease evolution, and to reconstruct unknown disease configurations at early stages from later measurements. In material engineering, they help determine internal properties of materials via non-destructive testing, detecting defects and optimising performance.

Traditional approaches, including variational, iterative, and Bayesian methods, rely on regularisation techniques to mitigate instability and noise. These methods use prior knowledge to stabilise solutions. Recently, data-driven techniques, particularly machine learning and deep learning, have revolutionised imaging applications, offering faster and more accurate reconstructions. Combining generative models with classical iterative methods, such as the Landweber scheme, preserves the explainability of inverse problem theory. However, despite impressive results, theoretical questions remain, especially concerning stability.

Despite these advances, numerous challenges persist. This special session will explore recent theoretical and numerical developments in inverse problems, emphasising applications in biomedicine and material engineering. Topics will include both knowledge-driven and data-driven methods, highlighting their respective strengths and limitations. Bringing together experts, this session aims to foster interdisciplinary collaboration and advance the field, improving the effectiveness of emerging technologies in healthcare and engineering.