NEURAL NETWORKS FOR SOLVING PDEs

A.J. OMELLA^{*}, J.M. TAYLOR[†] C. URIARTE ^{††}

Dpto. Matemática Aplicada, Universidad de Zaragoza María de Luna 3, 50018 Zaragoza, Spain e-mail: aomella@unizar.es

† Department of Mathematics, CUNEF Universidad Calle Almansa 101, 28040 Madrid, Spain e-mail: jamie.taylor@cunef.edu

†† BCAM – Basque Center for Applied Mathematics Alameda Mazarredo 14, 48009 Bilbao, Spain email: curiarte@bcamath.org

ABSTRACT

Neural networks have emerged as a robust alternative and a powerful complement to classical numerical schemes for solving Partial Differential Equations (PDEs). Their inherent capability to approximate complex solution spaces makes them ideal for tackling high-dimensional problems, discretization-invariant operator learning, and inverse problems, which are often intractable for traditional numerical approaches.

The primary objective of this thematic session is to disseminate recent progress in the fundamental mathematical and computational aspects of these techniques. The session will focus on the development of numerical methodology, convergence theory, and algorithmic efficiency.

The Topics of Interest include, but are not limited to:

- Fundamental Methods: Physics-Informed Neural Networks (PINNs), Deep Ritz methods, Deep First-Order System Least Squares, and operator learning approaches such as Fourier Neural Operators and DeepONets.
- Hybrid Strategies: Combining neural networks with classical methods (FEM or Boundary Elements).
- Mathematical Analysis: Convergence, error analysis, and optimization methodologies.
- Parametric PDEs: Applications in uncertainty quantification and inverse problems.

By emphasizing theoretical and methodological developments, we aim to establish a rigorous discussion about the capabilities, open challenges, and future research directions of neural networks for solving PDEs.