

## HYPERBOLIC APPROXIMATIONS OF HIGH-ORDER PDES: RECENT ADVANCES AND APPLICATIONS

FIRAS DHAOUADI\*, CARLOS MUNOZ MONCAYO#, HIROAKI NISHIKAWA†,  
MARIO RICCHIUTO+

\* Bordeaux INP, INRIA, Univ. Bordeaux, CNRS, IMB, UMR 5251, F-33400, Talence, France  
[firas.dhaouadi@inria.fr](mailto:firas.dhaouadi@inria.fr)

# King Abdullah University of Science and Technology, 23955, Thuwal, Saudi Arabia  
[carlos.munozmoncayo@kaust.edu.sa](mailto:carlos.munozmoncayo@kaust.edu.sa)

† National Institute of Aerospace, Hampton, VA 23666  
[hiroaki.nishikawa@nianet.org](mailto:hiroaki.nishikawa@nianet.org)

+ INRIA, Univ. Bordeaux, Bordeaux INP, CNRS, IMB, UMR 5251, F-33400, Talence, France  
[mario.ricchiuto@inria.fr](mailto:mario.ricchiuto@inria.fr)

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### ABSTRACT

Several applications in computational fluids dynamics involve partial differential equations (PDEs) with derivatives of order higher than one. A typical example is given by the Navier-Stokes equations, but many others exist as the Navier-Stokes-Korteweg system accounting for surface tension effects, the Green-Naghdi equations for dispersive free surface waves, and many others with applications in areas such as the modeling of blood flow, water waves, heat transfer, relativistic mechanics etc. In recent years, a substantial body of work has focused on approximating high-order PDE models by first-order hyperbolic systems. The motivations for this choice vary across fields: e.g., reduced numerical stiffness, improved gradient accuracy on irregular grids. This approach raises several important questions:

- How can hyperbolic reformulations improve the analysis, the understanding and the numerical approximation of high-order PDEs?
- Can we adapt classical numerical schemes and design new discretizations that fully leverage these hyperbolic formulations?
- How should initial and boundary conditions be imposed for the additional variables?
- Can these formulations offer practical advantages — or even enable problems that cannot be tackled with the original models — despite introducing additional equations and unknowns?

This mini-symposium aims to highlight recent advances in this rapidly growing area, bringing together analytical, numerical, and application-oriented perspectives.