

FINITE ELEMENT METHODS IN FLUID MECHANICS

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ABSTRACT

This minisymposium aims to bring together researchers across disciplines who use finite element methods to solve problems in fluid mechanics. Finite element methods are among many numerical techniques and algorithms regularly utilized to solve partial differential equations arising from continuum problems in fluid mechanics [1]. Their flexibility in terms of locally discretizing geometries, their versatility to be modified to specific properties of the underlying equations [2], and, in many cases, the applicability of rigorous mathematical theory to prove the effectiveness of the resulting algorithms, makes them an invaluable tool for the practical solution of both steady and dynamical fluid mechanics problems of physical and engineering interest [1,2,3].

Examples of problems where finite elements are typically used are those governed by the (linear) Stokes equations, which are accompanied by a rich mathematical theory of mixed finite elements, leading to the use of Taylor-Hood elements [4], Scott-Vogelius elements [5], and many others [3]. Other notable examples include those governed by the shallow water equations (both linearized and nonlinear) [9], as well as a suite of nonlinear partial differential equations like the celebrated incompressible Navier-Stokes equations [3], and compressible flow and other convection-dominated phenomena, relevant in aerodynamics, thermal transport, atmospheric and ocean science [8,10]. Novel non-conforming methods, such as discontinuous Galerkin methods (DG), hybridizable DG (HDG) among others, have also found several applications due to their suitable properties such as their hp adaptivity and computational efficiency. For instance, application in convection dominated problems [7], gas dynamics [14], compressible [11,15] and incompressible [16] flows. More recently, the use of finite element methods has also been observed in problems related to fluid stability, where typically spectral methods had been utilized [12,13].

The minisymposium is open to contributions ranging from the theoretical – the design of novel finite element discretizations, like SUPG, mixed FEM, DG, HDG, etc., or the proof of convergence of a particular numerical scheme, formulation or algorithm – to the more practical and computational – the computational verification of a new numerical scheme, their implementation in a high-performance computational architecture, structure-preserving

discretizations, or simply examples of the use of finite element methods in complex problems of engineering or scientific interest.

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PROSPECTIVE SPEAKERS:

- Felipe Lepe
- Gonzalo Rivera
- Jesús Vellojín
- Wilmar Imbachí
- Christian Nuñez
- Danilo Aballay
- Vicente Iligaray
- Gabriela Armentano
- Ingeborg Gjerde
- Ridgway Scott
- Tan Bui-Thanh
- Philip Lederer