Positivity-preserving discretisations without mesh restrictions

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ABSTRACT

In this talk I will review recent developments a method that respect the positivity of the discrete solution regardless of the geometry of the mesh and the order of the finite element method. The method, recently presented in [1], is built by first defining an algebraic projection onto the convex closed set of finite element functions that satisfy the bounds given by the solution of the PDE. Then, this projection is hardwired into the definition of the method by writing a discrete problem posed for this projected part of the solution. Since this process is done independently of the shape of the basis functions, and no result on the resulting finite element matrix is used, this process guarantees bound-preservation independently of the underlying mesh.

After explaining the main ideas, I will briefly review the application to steady-state [2] and time-dependent convection-diffusion equations, and the extension to the interesting case of polyhedral meshes. The final part of the talk will present some more recent developments in this area, focusing on the application to tensor-valued partial differential equations, with an emphasis on non-Newtonian fluid mechanics.

The results in this talk have been carried out in collaboration with Abdolreza Amiri (Strathclyde, UK), Ben Ashbi (Bath, UK), Emmanuil Geourgoulis (Heriot-Watt, UK and Athens, Greece), Tristan Pryer (Bath, UK), and Andreas Veeser (Milan, Italy).

REFERENCES

[1] G.R. Barrenechea, E. Georgoulis, T. Pryer, and A. Veeser, A nodally bound-preserving finite element method. IMA Journal on Numerical Analysis, 44 (4), 2198-2219, (2024).

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