

# Phase-Field Models of Binary Fluids in (Soft-)Wetting

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## ABSTRACT

Binary fluids are fluids that comprise two constituents, viz. two phases of the same fluid (gas or liquid) or two distinct species (e.g. water and air). A distinctive feature of binary-fluids is the presence of a fluid-fluid interface that separates the two components. This interface generally carries surface energy and accordingly it introduces capillary forces. The interaction of a binary-fluid with a (possibly deformable) solid engenders a variety of intricate physical phenomena. The dynamics of the contact line which occurs at the intersection of the fluid-fluid interface with the solid surface is a classical but notoriously difficult problem in fluid mechanics, and despite significant advancements [1-3] in the theory, contemporary understanding of such dynamic wetting is still incomplete.

The complexity of wetting dynamics is significantly aggravated if the solid substrate is elastic. The interaction of a binary-fluid with a soft solid substrate is generally referred to as *elastocapillarity* or *soft wetting*. Soft wetting is positioned at the crossroads of several important areas in physics and mechanics, viz. soft matter, complex fluids, and fluid structure interaction. Soft wetting underlies miscellaneous complex physical phenomena such as durotaxis [4] and capillary origami [5], and it is of fundamental technological relevance in a wide variety of high-tech industrial applications, such as inkjet printing and additive manufacturing.

Phase-field models of binary fluids permit a comprehensive description of complex wetting phenomena in geometrically complex scenarios [6,7]. Accordingly, phase-field models of binary fluids are ideally suited for modeling dynamic (soft-)wetting problems [6,8-11].

In this presentation, we consider recent developments in computational modeling of dynamic wetting and soft wetting based on a phase-field model for the binary fluid. The phase-field binary-fluid model is described by the incompressible Navier–Stokes–Cahn–Hilliard equations [12] with preferential-wetting and generalized Navier boundary conditions at the fluid-solid interface [6]. To resolve the fluid-fluid interface and the localized displacements in the solid, we apply adaptive hierarchical spline approximations [6,9]. We regard several aspects of the formulation and of the simulation techniques.

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