Adaptive sampling and surrogate/reduced order modelling strategies for parametric differential equations

BENJAMIN M. KENT^{*}, LORENZO TAMELLINI^{*}

*CNR-IMATI Via Ferrata 5/a, 27100 Pavia, Italy kent@imati.cnr.it, tamellini@imati.cnr.it

ABSTRACT

Parametric differential equations are ubiquitous in science and engineering. Being able to efficiently explore the parameter space and build approximations of the parametric dependence of the solution is essential in many fields, such as uncertainty quantification, optimization, design of experiments, and real-time control of systems.

Dealing with such equations typically boils down to repeatedly solving them for different values of the parameters. However, since differential equations are typically expensive to solve, it is crucial to keep evaluations to a minimum, which naturally leads to using adaptive strategies. When it comes to sampling, these might be e.g. advanced MCMC sampling, importance sampling, or control variates methods. In the context of surrogate model construction, adaptivity can enter in virtually any step of the procedure: selection of suitable basis functions (e.g. reduced basis snapshots), hyperparameter calibration (e.g. rank of low-rank approximation or shape parameters for basis functions), dimension-adaptivity (i.e., refining along the dimensions of the parameter space that are deemed more important), and local adaptivity (i.e., refining the regions of the parameter spaces in which local features are detected). Moreover, both sampling and surrogate model construction can benefit from dimensionality reduction techniques (either pruning away parameters that are found to be negligible, or coming up with new coordinate systems that are aligned with the directions of maximal variability), multi-fidelity approaches (in which the computational budget is gradually allocated over a pool/hierarchy of approximations of the differential equation), and of course, mesh-adaptivity/adaptive-time-stepping for the solvers at hand. Adaptivity plays an important role also in data-assimilation problems, in which the computational model needs to adjust to newly acquired data as they become available.

Both methodological and application-oriented contributions in this framework are welcome to this minisymposium.